

Questions Related to Session #5 Regression and Correlation

Study

Forty four males and 44 females were randomly assigned to treadmill workouts which lasted from 306 to 976 seconds. VO₂ Max (maximum O₂ consumption normalized by body weight (ml/kg.min)) was the outcome measure.

Regression Model 1

The following common slope multiple linear regression model was estimated by least squares.

$$E(\text{VO}_2 \text{ Max}_i | X) = \beta_0 + \beta_1(\text{exercise duration}_i) + \beta_2(z_{2,i})$$

where $z_{2,i} = 1$ if the i th participant was male, and 0 if i th participant was female.

Regression Analysis Summary

Table 1. The regression ANOVA table from the multiple regression analysis.

Parameter	DF	SS	MS	F _{observed}
Regression	2	6044.03		
Error	84	1335.60		
Total	86	7379.62		

Table 2. The regression parameter estimates.

Parameter	Estimate b	SE b	t _{observed}
Intercept	-1.360	2.220	
Duration	0.059	0.004	
Gender=male	3.396	1.016	

- 1) Utilize the information from Table 1 to compute the mean square regression (MSR), the mean square error (MSE) and the F-statistic (F_{observed}).
- 2) For a two-sided test with significance level $\alpha=0.05$ we reject $H_0: \beta_1=\beta_2=0$, if $F_{\text{observed}} \geq F_{(2,84,.95)} = 3.105$. Do we reject?
- 3) Utilizing the information from Table 1, compute the value of the coefficient of determination (R^2), and give a simple interpretation for the R^2 value you calculated.

- 4) Utilizing the information from Table 2, compute the t -statistic (t_{observed}) for the regression parameter related to exercise duration and for the regression parameter related to gender=male.
- 5) For a two-sided test with significance level 0.05 we reject $H_0: \beta_1=0$, and $H_0: \beta_2=0$ if $|t_{\text{observed}}| \geq t_{(84, .975)}=1.990$. Do we reject $H_0: \beta_1=0$?. Do we reject $H_0: \beta_2=0$?.
- 6) For females and males, write out the estimated regression equation for predicting the expected value $VO_2 \text{ Max}$ as a linear function of exercise duration.
- 7) What is the expected value of $VO_2 \text{ Max}$ for a female who spent 450 seconds on the treadmill?. What is the expected value of $VO_2 \text{ Max}$ for a male who spent 450 seconds on the treadmill.

Regression Model 2

The following separate slopes multiple linear regression model was fit to the same data by least squares.

$$E(\text{VO}_2 \text{ Max}_i | X) = \beta_0 + \beta_1(\text{exercise duration}_i) + \beta_2(z_{2,i}) + \beta_3(z_{2,i} \times \text{exercise duration}_i)$$

where $z_{2,i} = 1$ if the i th participant was male, and 0 if i th participant was female.

Regression Analysis Summary

Table 3. The regression ANOVA table from the multiple regression analysis.

Parameter	DF	SS	MS	F_{observed}
Regression	3	6089.35	2029.12	130.57
Error	83	1290.27	15.54	
Total	86	7379.62		

- 7) Utilizing the information from Table 1 and Table 3, compute the extra-sum of squares F-test for the null hypothesis $H_0: \beta_3 = 0$.
- 9) For a two-sided test with significance level $\alpha=0.05$ we reject $H_0: \beta_3=0$, if $F_{\text{observed}} \geq F_{(1,83, .95)} = 3.955$. Do we reject?.
- 10) We observe from a sample of 44 paired measurements a sample correlation $r = 0.35$. Based on this information compute the value of the one-sample t-Test (t_{observed}).
- 11) For a two-sided test with significance level $\alpha=0.05$ we reject $H_0: \rho = 0$ if $|t_{\text{observed}}| \geq t_{(42,0.975)} = 2.08$. Do we reject?.

